

$$\begin{cases} x^2 y'' + 2x y' - 2y = x + 4 & x > 0 \\ y(1) = 0 \\ y'(1) = 1 \end{cases}$$

- $x^2, 2x, -2, x, 4$ son cont. en $x > 0$
- $x^2 \neq 0 \forall x \in x > 0$
- 2. Cond. Iniciales en $x = 1 > 0$

sol. y es única en $x > 0$

$$\downarrow x = e^t \Rightarrow t = \ln x$$

$$y''(t) + y'(t) - 2y(t) = e^t + 4 \Rightarrow m^2 + m - 2 = 0 \Rightarrow m = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$$y_c(t) = C_1 e^t + C_2 e^{-2t}$$

$$e^t \Rightarrow \{e^t\} \Rightarrow \{t e^t\}$$

$$\Rightarrow y_p(t) = A t e^t + B$$

$$y_p'(t) = A e^t + A t e^t$$

$$y_p''(t) = A e^t + A e^t + A t e^t = 2A e^t + A t e^t$$

$$\Rightarrow 2A e^t + A t e^t + A e^t + A t e^t - 2(A t e^t + B) = e^t + 4$$

$$3A e^t - 2B = e^t + 4 \Rightarrow 3A = 1 \quad -2B = 4$$

$$y(t) = C_1 e^t + C_2 e^{-2t} + \frac{1}{3} t e^t - 2 \quad \left\{ \begin{array}{l} \text{de hacer} \\ \text{cambio} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} A = \frac{1}{3} \\ B = -2 \end{array} \right. \Rightarrow y_p(t) = \frac{1}{3} t e^t - 2$$

$$y(x) = C_1 x + C_2 x^{-2} + \frac{1}{3} (\ln x) x - 2$$

$$y(1) = 0 \Rightarrow C_1 + C_2 - 2 = 0 \Rightarrow C_1 + C_2 = 2$$

$$y'(x) = C_1 - 2C_2 x^{-3} + \frac{1}{3} (1 + \ln x) \Rightarrow y'(1) = 1 \Rightarrow C_1 - 2C_2 + \frac{1}{3} = 1 \Rightarrow C_1 - 2C_2 = \frac{2}{3}$$

$$\Rightarrow C_1 - 2C_2 = \frac{2}{3} \quad \begin{cases} C_1 = 2 \\ C_2 = 4 \end{cases}$$

$$C_1 + C_2 = 2 \Rightarrow y(x) = -2x + 4x^{-2} + \frac{1}{3} x \ln(x) - 2$$